How to Solve the Zebra Problem

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Problem Formulation

There are five houses of different colours, inhabited by different nationals, with different pets, drinks, and sports.

Furthermore, there are the following 14 additional constraints which I have changed for the occasion:

1. The Englishman lives in the red house.

2. The Spaniard owns a dog.

3. The man in the green house drinks coffee.

4. The Irishman drinks tea.

5. The green house is to the right of the ivory house.
6. The Go player owns snails.

7. The man in the yellow house plays cricket.

8. The guy in the house int he middle drinks milk.

9. The Nigerian lives in the first house.¹

10. The judo player lives next to the man who has a fox.

11. The cricketer lives next to the man who has a horse.

12. The poker player drinks orange juice.


¹Originally, this was “at the end” but this would make the problem too easy.
14. The Nigerian lives next to the blue house.
14. The Nigerian lives next to the blue house.

The question is:
14. The Nigerian lives next to the blue house.

The question is: **Who owns the zebra**
14. The Nigerian lives next to the blue house.

The question is: **Who owns the zebra and who drinks Guinness?**
According to folklore, the Zebra Problem was designed by the English logician Charles Lutwidge Dodgson (a.k.a. Lewis Carroll. Born: 27 Jan 1832, Died: 14 Jan 1898).
Some History

According to folklore, the Zebra Problem was designed by the English logician Charles Lutwidge Dodgson (a.k.a. Lewis Carroll. Born: 27 Jan 1832, Died: 14 Jan 1898). I don’t have a reference. If you do, then please let me know.
Modeling the Problem

We can model the problem as a CSP. We number the houses (left to right) from 1 to 5. We then assign houses to things and we reduce the problem to the following:

- The number assigned to the person who drinks Guinness is the same as the number assigned to Guinness;
- The number assigned to the person who owns the zebra is the same as the number assigned to the zebra.
nationalities Englishman = \( A_1 \), Spaniard = \( A_2 \), Irishman = \( A_3 \), Nigerian = \( A_4 \), Japanese = \( A_5 \).

plays go = \( B_1 \), cricket = \( B_2 \), judo = \( B_3 \), poker = \( B_4 \), polo = \( B_5 \).

drinks coffee = \( C_1 \), tea = \( C_2 \), milk = \( C_3 \), orange juice = \( C_4 \), Guinness = \( C_5 \),

pets dog = \( D_1 \), snails = \( D_2 \), fox = \( D_3 \), horse = \( D_4 \), zebra = \( D_5 \).

colours red = \( E_1 \), green = \( E_2 \), ivory = \( E_3 \), yellow = \( E_4 \), blue = \( E_5 \).
If $X$ is a letter then $X_i \neq X_j \iff i \neq j$. Furthermore, we have:

1. The Englishman ($A_1$) lives in the red ($E_1$) house: $A_1 = E_1$.
2. The Spaniard ($A_2$) owns a dog ($D_1$): $A_2 = D_1$.
3. The man in the green ($E_2$) house drinks coffee ($C_1$): $E_2 = C_1$.
4. The Irishman ($A_3$) drinks tea ($C_2$): $A_3 = C_2$.
5. The green ($E_2$) house is to the right of the ivory ($E_3$) house: $E_2 - E_3 = 1$.
6. The Go ($B_1$) player owns snails ($D_2$): $B_1 = D_2$.
7. The man in the yellow ($E_4$) house plays cricket ($B_2$): $E_4 = B_2$. 
8. The guy in the house in the middle drinks milk ($C_3$): $C_3 = 3$.


10. The Judo ($B_3$) player lives next to the man who has a fox ($D_3$): $|B_3 - D_3| = 1$.

11. The cricketer ($B_2$) lives next to the man who has a horse ($D_4$): $|B_2 - D_4| = 1$.

12. The poker ($B_4$) player drinks orange juice ($C_4$): $B_4 = C_4$.

13. The Japanese ($A_5$) plays polo ($B_5$): $A_5 = B_5$.

14. The Nigerian ($A_4$) lives next to the blue ($E_5$) house: $|A_4 - E_5| = 1$. 
Initial CSP.
Because of the **unary** constraints (Rules 8 and 9) \( A_4 \) must be 1 and \( C_3 \) must be 3.
We can remove the red values.

\[ \begin{array}{c}
A_1 & \text{Englishman} \\
A_2 & \text{Spaniard} \\
A_3 & \text{Irishman} \\
A_4 & \text{Nigerian} \\
A_5 & \text{Japanese} \\
B_1 & \text{go} \\
B_2 & \text{cricket} \\
B_3 & \text{judo} \\
B_4 & \text{poker} \\
B_5 & \text{polo} \\
C_1 & \text{coffee} \\
C_2 & \text{tea} \\
C_3 & \text{milk} \\
C_4 & \text{orange juice} \\
C_5 & \text{Guinness} \\
D_1 & \text{dog} \\
D_2 & \text{snails} \\
D_3 & \text{fox} \\
D_4 & \text{horse} \\
D_5 & \text{zebra} \\
E_1 & \text{red} \\
E_2 & \text{green} \\
E_3 & \text{ivory} \\
E_4 & \text{yellow} \\
E_5 & \text{blue} \\
\end{array} \]
The resulting CSP is called **Node-Consistent**.
Some Values have no Support.

A_1 Englishman
A_2 Spaniard
A_3 Irishman
A_4 Nigerian
A_5 Japanese
B_1 go
B_2 cricket
B_3 judo
B_4 poker
B_5 polo
C_1 coffee
C_2 tea
C_3 milk
C_4 orange juice
C_5 Guinness
D_1 dog
D_2 snails
D_3 fox
D_4 horse
D_5 zebra
E_1 red
E_2 green
E_3 ivory
E_4 yellow
E_5 blue
Let’s mark them for removal and colour them red.
If we **propagate** the consequences of removing the red values, more values will lose support.
Remove

A_1  Englishman
A_2  Spaniard
A_3  Irishman
A_4  Nigerian
A_5  Japanese
B_1  go
B_2  cricket
B_3  judo
B_4  poker
B_5  polo
C_1  coffee
C_2  tea
C_3  milk
C_4  orange juice
C_5  Guinness
D_1  dog
D_2  snails
D_3  fox
D_4  horse
D_5  zebra
E_1  red
E_2  green
E_3  ivory
E_4  yellow
E_5  blue
and Propagate.
More Propagation.

A_1  Englishman
A_2  Spaniard
A_3  Irishman
A_4  Nigerian
A_5  Japanese
B_1  go
B_2  cricket
B_3  judo
B_4  poker
B_5  polo
C_1  coffee
C_2  tea
C_3  milk
C_4  orange juice
C_5  Guinness
D_1  dog
D_2  snails
D_3  fox
D_4  horse
D_5  zebra
E_1  red
E_2  green
E_3  ivory
E_4  yellow
E_5  blue
More Values will lose Support.
More Propagation...
We have reached a Fix-point
The resulting CSP is called **Arc-consistent**.
$E_4$’s domain contains 1.
The domains of the other $E_i$ do not contain 1.
$E_4$ must be $1$. 

<table>
<thead>
<tr>
<th>A₁</th>
<th>Englishman</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₂</td>
<td>Spaniard</td>
</tr>
<tr>
<td>A₃</td>
<td>Irishman</td>
</tr>
<tr>
<td>A₄</td>
<td>Nigerian</td>
</tr>
<tr>
<td>A₅</td>
<td>Japanese</td>
</tr>
<tr>
<td>B₁</td>
<td>go</td>
</tr>
<tr>
<td>B₂</td>
<td>cricket</td>
</tr>
<tr>
<td>B₃</td>
<td>judo</td>
</tr>
<tr>
<td>B₄</td>
<td>poker</td>
</tr>
<tr>
<td>B₅</td>
<td>polo</td>
</tr>
<tr>
<td>C₁</td>
<td>coffee</td>
</tr>
<tr>
<td>C₂</td>
<td>tea</td>
</tr>
<tr>
<td>C₃</td>
<td>milk</td>
</tr>
<tr>
<td>C₄</td>
<td>orange juice</td>
</tr>
<tr>
<td>C₅</td>
<td>Guinness</td>
</tr>
<tr>
<td>D₁</td>
<td>dog</td>
</tr>
<tr>
<td>D₂</td>
<td>snails</td>
</tr>
<tr>
<td>D₃</td>
<td>fox</td>
</tr>
<tr>
<td>D₄</td>
<td>horse</td>
</tr>
<tr>
<td>D₅</td>
<td>zebra</td>
</tr>
<tr>
<td>E₁</td>
<td>red</td>
</tr>
<tr>
<td>E₂</td>
<td>green</td>
</tr>
<tr>
<td>E₃</td>
<td>ivory</td>
</tr>
<tr>
<td>E₄</td>
<td>yellow</td>
</tr>
<tr>
<td>E₅</td>
<td>blue</td>
</tr>
</tbody>
</table>
After Assignment $E_4 = 1$ and Arc-Consistency.
$C_5$’s domain contains 1.
The domains of the other $C_i$ do not contain 1.
$C_5$ must be 1.
After Assignment $C_5 = 1$ and Arc-Consistency.
Start **MAC-Search** (Maintain Arc-Consistency).
Select $C_1$ as Current Variable.
After Assignment $C_1 = 4$ and Arc-Consistency.
Select $B_1$ as Current Variable.
After Assignment \( B_1 = 3 \) and Arc-Consistency.
All domains are empty. We must backtrack on $B_1$. 

$A_1$ Englishman  
$A_2$ Spaniard  
$A_3$ Irishman  
$A_4$ Nigerian  
$A_5$ Japanese  
$B_1$ go  
$B_2$ cricket  
$B_3$ judo  
$B_4$ poker  
$B_5$ polo  
$C_1$ coffee  
$C_2$ tea  
$C_3$ milk  
$C_4$ orange juice  
$C_5$ Guinness  
$D_1$ dog  
$D_2$ snails  
$D_3$ fox  
$D_4$ horse  
$D_5$ zebra  
$E_1$ red  
$E_2$ green  
$E_3$ ivory  
$E_4$ yellow  
$E_5$ blue
Next Assignment to $B_1$. 

$A_1$ Englishman
$A_2$ Spaniard
$A_3$ Irishman
$A_4$ Nigerian
$A_5$ Japanese
$B_1$ go
$B_2$ cricket
$B_3$ judo
$B_4$ poker
$B_5$ polo
$C_1$ coffee
$C_2$ tea
$C_3$ milk
$C_4$ orange juice
$C_5$ Guinness
$D_1$ dog
$D_2$ snails
$D_3$ fox
$D_4$ horse
$D_5$ zebra
$E_1$ red
$E_2$ green
$E_3$ ivory
$E_4$ yellow
$E_5$ blue
After Assignment $B_1 = 4$ and Arc-Consistency.
We must backtrack on $C_1$. 

$A_1$ Englishman
$A_2$ Spaniard
$A_3$ Irishman
$A_4$ Nigerian
$A_5$ Japanese
$B_1$ go
$B_2$ cricket
$B_3$ judo
$B_4$ poker
$B_5$ polo
$C_1$ coffee
$C_2$ tea
$C_3$ milk
$C_4$ orange juice
$C_5$ Guinness
$D_1$ dog
$D_2$ snails
$D_3$ fox
$D_4$ horse
$D_5$ zebra
$E_1$ red
$E_2$ green
$E_3$ ivory
$E_4$ yellow
$E_5$ blue
Next Assignment to $C_1$.
After Assignment $C_1 = 5$ and Arc-Consistency.
Select $B_4$ as Current Variable.
After Assignment $B_4 = 2$ and Arc-Consistency.
Backtrack on $B_4$. 
Next Assignment to $B_4$. 

$A_1$ Englishman
$A_2$ Spaniard
$A_3$ Irishman
$A_4$ Nigerian
$A_5$ Japanese
$B_1$ go
$B_2$ cricket
$B_3$ judo
$B_4$ poker
$B_5$ polo
$C_1$ coffee
$C_2$ tea
$C_3$ milk
$C_4$ orange juice
$C_5$ Guinness
$D_1$ dog
$D_2$ snails
$D_3$ fox
$D_4$ horse
$D_5$ zebra
$E_1$ red
$E_2$ green
$E_3$ ivory
$E_4$ yellow
$E_5$ blue
After Assignment $B_4 = 4$ and Arc-Consistency.
All domains are singletons.
All constraints are satisfied.
We have solved the problem.

A₁  Englishman
A₂  Spaniard
A₃  Irishman
A₄  Nigerian
A₅  Japanese
B₁  go
B₂  cricket
B₃  judo
B₄  poker
B₅  polo
C₁  coffee
C₂  tea
C₃  milk
C₄  orange juice
C₅  Guinness
D₁  dog
D₂  snails
D₃  fox
D₄  horse
D₅  zebra
E₁  red
E₂  green
E₃  ivory
E₄  yellow
E₅  blue
$D_5 = 5$ (the zebra).
\( D_5 = 5 \) (the zebra). \( A_5 = 5 \) (the Japanese).
$D_5 = 5$ (the zebra). $A_5 = 5$ (the Japanese). Therefore, the Japanese owns the zebra.
To some this may have come as a complete surprise.
$C_5 = 1$ (Guinness).
$C_5 = 1$ (Guinness). $A_4 = 1$ (the Nigerian).
$C_5 = 1$ (Guinness). $A_4 = 1$ (the Nigerian). Therefore, the Nigerian drinks Guinness.
Given that Lagos has a large Guinness brewery, this should not have come as a complete surprise.